African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30 Juin 2025

Analyse de l'efficience des marchés de taux de change de l'Afrique du nord: Approche Multifractale

Analysis of exchange rate markets efficiency in North Africa: Multifractal approach

Auteur 1 : BENBACHIR Soufiane

BENBACHIR Soufiane, (ORCID https://orcid.org/0009-0003-9357-9188, PhD)
Université Mohammed V-Rabat
Faculté des Sciences Juridiques, Economiques et Sociales-Agdal-Maroc

Déclaration de divulgation : **L'auteur n'a pas connaissance de quelconque financement qui pourrait affecter l'objectivité de cette étude.**

Conflit d'intérêts : **L'auteur ne signale aucun conflit d'intérêts.**

Pour citer cet article : BENBACHIR .S (2025). « Analyse de l'efficience des marchés de taux de change de l'Afrique du nord: Approche multifractale », African Scientific Journal « Volume 03, Numéro 30 » pp: 0676 – 0703.

DOI : 10.5281/zenodo.15745149

African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30, Juin 2025

Résumé

Cette étude examine les propriétés multifractales et l'efficience des marchés des changes marocain, algérien et tunisien en utilisant la méthode d'analyse multifractale des fluctuations sans tendance (MF-DFA). L'analyse révèle des preuves solides de comportements multifractals dans les trois marchés, caractérisés par des fluctuations dépendantes de l'échelle, des exposants généralisés de Hurst et de Rényi non linéaires, ainsi que des spectres de singularité en forme de parabole inversée. Ces résultats suggèrent des dynamiques temporelles complexes et hétérogènes, qui s'écartent des hypothèses de la forme faible de l'hypothèse d'efficience des marchés (EMH). Parmi les trois marchés, le Maroc présente le degré de multifractalité le plus élevé, et donc le plus faible niveau d'efficience, suivi de l'Algérie, tandis que la Tunisie apparaît comme le marché le plus efficient. Pour comprendre les sources de la multifractalité, l'étude compare les séries originales à des séries de substitution et des séries mélangées (shuffled), confirmant que les corrélations à long terme et les distributions à queues épaisses contribuent toutes deux aux à la multifractalité observée. Ces résultats mettent en évidence l'insuffisance des modèles linéaires basés sur des hypothèses gaussiennes pour modéliser la dynamique des taux de change dans ces marchés. Les implications pratiques sont importantes : les traders peuvent exploiter les régularités persistantes, et les décideurs politiques peuvent s'appuyer sur ces résultats pour corriger les inefficiences structurelles et renforcer la transparence. Une limite majeure de l'étude est l'exclusion de l'Égypte et de la Libye en raison de contraintes liées à la disponibilité des données. Les recherches futures devraient intégrer des données à haute fréquence et explorer les interdépendances multifractales entre marchés à l'aide de l'analyse croisée multifractale (MFDCCA).

Mots clés: Exchange rate markets, Multifractality**,** Generalized Hurst exponents, Rényi exponents, Singularity spectrum

African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30, Juin 2025

Abstract

This study examines the multifractal properties and efficiency of the Moroccan, Algerian, and Tunisian foreign exchange markets using the Multifractal Detrended Fluctuation Analysis (MF-DFA) method. The analysis reveals strong evidence of multifractal behavior in all three markets, characterized by scale-dependent fluctuations, nonlinear generalized Hurst and Rényi exponents, and inverted parabolic singularity spectra. These findings suggest complex and heterogeneous temporal dynamics that deviate from the assumptions of the Efficient Market Hypothesis (EMH) in its weak form. Among the markets, Morocco exhibits the highest degree of multifractality and thus the lowest efficiency, followed by Algeria, with Tunisia being the most efficient. To understand the sources of multifractality, the study compares original data with surrogate and shuffled series, confirming that both long-range correlations and heavy-tailed distributions contribute to the observed multifractal patterns. These results underscore the inadequacy of linear and Gaussian-based models in capturing exchange rate dynamics in these markets. The practical implications are significant: traders can exploit persistent patterns, and policymakers may use these insights to address structural inefficiencies and improve transparency. A key limitation is the exclusion of Egypt and Libya due to data constraints. Future research should incorporate high-frequency data and explore cross-market multifractal dependencies using Multifractal Detrended Cross-Correlation Analysis (MFDCCA).

Keywords: Exchange rate markets, Multifractality**,** Generalized Hurst exponents, Rényi exponents, Singularity spectrum

African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30, Juin 2025

Introduction

Exchange rate markets play a fundamental role in shaping a country's economic trajectory. They influence trade balances, capital flows, inflation, and overall macroeconomic stability. In North Africa, where economies differ in openness, resource dependence, and institutional maturity, the behavior of exchange rate markets carries important implications for monetary policy, foreign direct investment, and financial integration. Assessing the efficiency of these markets is therefore critical for investors, policymakers, and institutions seeking to make informed decisions and manage currency risk effectively.

The Efficient Market Hypothesis (EMH) (Fama, 1970), particularly in its weak form, posits that asset prices fully incorporate all available historical information, rendering future price movements unpredictable and following a random walk. If exchange rates in North African countries conform to this hypothesis, it would imply limited scope for speculation based on historical trends and affirm the reliability of market-based pricing mechanisms. However, developing and emerging markets often deviate from this ideal due to structural inefficiencies such as low market liquidity, regulatory interventions, political instability, and informational asymmetries.

Conventional econometric methods; including unit root tests, autocorrelation analysis, and variance ratio tests; have been widely used to test market efficiency. Yet, they often fail to capture the nonlinear dynamics, volatility clustering, and scaling behaviors characteristic of real-world financial time series. In contrast, multifractal models offer a more robust framework for analyzing such complexities. These models can detect long-range dependence, fat-tailed distributions, and multifractality, features commonly observed in financial markets but frequently overlooked by linear approaches. Among them, Multifractal Detrended Fluctuation Analysis (MF-DFA) has emerged as a particularly effective technique for quantifying the degree of multifractality and evaluating the scaling properties of time series data.

This study applies the MF-DFA methodology to assess the efficiency of exchange rate markets in selected North African countries, specifically Algeria, Morocco, and Tunisia. Although Egypt is one of the North African region's most significant economies, it is excluded from the analysis due to extended periods of fixed or nearly constant exchange rates under tight currency controls and managed pegs. These result in return series with minimal variability, which are not suitable for multifractal analysis, as the method requires sufficient price fluctuations to reveal meaningful scaling behavior. Libya is also excluded, primarily due to severe political instability, a fragmented monetary system, and the existence of multiple exchange rate regimes,

African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30, Juin 2025

official and unofficial, leading to inconsistencies and data unreliability. These issues make it difficult to derive a continuous, market-driven exchange rate series suitable for empirical analysis.

Using daily exchange rate returns against US dollar, this study seeks to identify the presence and sources of multifractality, thereby evaluating the degree of market inefficiency. In particular, it investigates whether multifractal properties arise from long-range temporal correlations, fat-tailed return distributions, or both. The findings are intended to provide a deeper understanding of the dynamic behavior of exchange rate markets in North Africa against US dollar and contribute to the broader literature on financial market efficiency in developing economies.

In light of this, the research is guided by the following key questions:

- To what extent are exchange rate markets in North African countries efficient in the weak-form sense?
- Do the exchange rate return series exhibit multifractal behavior, and what are the primary sources of this multifractality?
- How do these characteristics differ across countries in the region?

The remainder of this paper is organized as follows. Section 1 reviews the existing literature on exchange rate market efficiency and multifractal analysis. Section 2 presents the dataset and details the methodological approach, with a focus on the MF-DFA technique. Section 3 reports the empirical results and provides an interpretation and discussion of the findings. The study is concluded with a summary of results, a discussion of practical implications, and a suggestion of avenues for future research.

1. Literature review

The following literature review examines empirical studies that assess foreign exchange market efficiency using both traditional and multifractal approaches. To begin with, several studies have tested the Efficient Market Hypothesis using classical methods. For instance, Bogdan and Miloş (2009) examined the Efficient Market Hypothesis (EMH) in the context of the Bucharest Stock Exchange, focusing on whether stock prices fully reflect available information. In addition to reviewing key theoretical and empirical literature on EMH, the authors conducted an empirical test using daily time series data of the BET index from 2000 to 2009. The econometric analysis found evidence supporting the weak form of market efficiency, indicating

African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30, Juin 2025

that past price information is already reflected in current prices and cannot be used to generate consistent excess returns.

Similarly, Giannellis and Papadopoulos (2009) proposed an alternative method to test FOREX market efficiency in developing countries, based on whether actual exchange rates significantly deviate from their equilibrium values. Using a logistic smooth transition autoregressive (LSTAR) model and linear unit root tests, the study examined the Poland/Euro, Czech/Euro, and Slovak/Euro exchange markets. The results showed no evidence of nonlinear adjustment, leading to the conclusion that the Poland/Euro market is efficient, the Czech/Euro market is inefficient, and the Slovak/Euro market is quasi-efficient.

Moreover, Abounoori et al (2012) tested the weak-form Efficient Market Hypothesis (EMH) in the Iranian Rial/US Dollar exchange rate using detrended fluctuation analysis (DFA) on data from December 2005 to April 2010. The analysis, divided into four subperiods, found that the Iranian forex market is weak-form inefficient throughout the full sample and in each subperiod. However, the degree of inefficiency varied over time, indicating that past data could be used to make profitable, risk-adjusted trades, thereby contradicting weak-form EMH.

In addition, Çıtak et al (2015) tested the weak-form Efficient Market Hypothesis (EMH) for the Turkish foreign exchange market using weekly TRY/USD exchange rate data from January 2000 to December 2013. Applying ADF and PP unit root tests, Lo and MacKinlay's variance ratio test, and the Ljung-Box Q test, the study rejected the random walk hypothesis, which indicates that the Turkish exchange rate market does not exhibit weak-form efficiency.

Furthermore, El Abed and Maktouf (2016) examined the asymmetry and long memory in East Asian exchange rates expressed in US dollars using the FIAPARCH model over the period January 1, 1999 to September 30, 2015. The study found evidence of asymmetric effects in exchange rate returns and high persistence, indicating long-range dependence in these markets. The results highlight significant volatility dynamics relevant to international investors and policymakers.

Likewise, Amelot et al (2017) investigated the efficiency of the Mauritian foreign exchange market under the Efficient Market Hypothesis (EMH) framework, analyzing daily nominal spot rates for EUR/MUR, USD/MUR, GBP/MUR, and JPY/MUR from 2012 to 2016. To test weak-form efficiency, the study employed Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests, which confirmed that exchange rates follow a random walk, thus supporting weak-form efficiency. For semi-strong form efficiency, the Johansen Cointegration Test, Granger Causality Test, and Variance Decomposition were applied. The results revealed no

African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30, Juin 2025

long-run relationships, but unidirectional and bidirectional causalities and long-term co-movements among exchange rates, indicating predictability and hence a rejection of semi-strong form efficiency. Consequently, the study concluded that the Mauritian forex market is weak-form efficient but inefficient in the semi-strong form.

In a similar vein, Kallianiotis (2017) examined the efficiency of the foreign exchange market by analyzing four major exchange rates: USD/EUR, USD/GBP, CAD/USD, and JPY/USD. The study applied various theoretical models, including the Random Walk Hypothesis, Unbiased Forward Rate Hypothesis, Composite Efficiency Hypothesis, Semi-Strong Efficiency, and models based on anticipated and unanticipated news. A GARCH (p, q) model was used to estimate the exchange rate risk premium. The results showed that while relative market efficiency exists, significant risk premia are present for some currencies, suggesting partial inefficiency in certain segments of the forex market.

Moreover, Kumar (2018) tested the Adaptive Market Hypothesis (AMH) for Indian exchange rates against the USD, GBP, Euro, and Yen using the automatic variance ratio test and rank-based tests by Belaire-Franch and Contreras (2004). By applying overlapping and non-overlapping moving subsample approaches, the study examined the time-varying nature of market efficiency. Results showed violations of the martingale hypothesis for the USD and Yen over the full sample, and revealed that the predictability of exchange rate returns fluctuates over time, especially during major macroeconomic events. These findings therefore support the validity of the Adaptive Market Hypothesis in the Indian foreign exchange market.

Similarly, Njindan (2019) investigated the efficiency of Indonesia's foreign exchange (FX) market, which was important for investors and policymakers. Traditional tests for market efficiency often assumed independent, identically distributed errors and treated structural breaks as external factors, potentially biasing results. To improve accuracy, the study employed a GARCH-based unit root test that accounted for both heteroskedasticity and endogenous structural breaks. The analysis found up to two structural breaks in Indonesian exchange rates and that the error terms were not i.i.d. Conventional tests rejected market efficiency about 29% of the time, but this rejection rate rose to 50% when heteroskedasticity and structural breaks were considered, indicating greater inefficiency. These results held across daily data and different time periods, with the market being less efficient before the Asian Financial Crisis. Additionally, the study estimated that 71% of exchange rates reverted to the mean within one month, suggesting short-term efficiency. Overall, Indonesia's FX market had improved but

African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30, Juin 2025

remained partially inefficient, implying opportunities for profit and the need for enhanced regulatory transparency.

Furthermore, Sashikanta and Pattanayak (2020) empirically examined the time-varying efficiency of the Indian foreign exchange market (INR–USD) within the framework of the Adaptive Market Hypothesis (AMH). The study aimed to determine whether market efficiency was static or evolved over time, and to identify key events responsible for any observed variations. Using robust statistical methods and a fixed-length rolling window approach to avoid data-snooping bias, the authors found that market efficiency was not constant but fluctuated in response to changing market conditions. Episodes of efficiency and inefficiency coincided with major economic and institutional events, such as changes in exchange rate regimes, financial turbulence, central bank interventions, and shifts in trade volume. Thus, these findings supported the AMH and suggested that the Indian FX market alternated between efficient and inefficient states, thereby creating potential arbitrage opportunities for traders during periods of inefficiency.

In addition, Namhoon et al (2021) examined overreaction in the foreign exchange market using various combinations of formation and testing periods over a 30-year span. They found that return reversals were more significant when both the formation and test periods were longer. However, there was no consistent evidence of persistent momentum or reversal across the entire sample period, which suggests that overreaction results are sensitive to the chosen time frame. Additionally, portfolios of loser currencies generally outperformed, except in the short term when spot rates were used. Overall, the evidence of overreaction strengthened with longer periods, highlighting the importance of time horizon in analyzing FX market behavior.

Similarly, Benzai et al (2022) examined the Efficient Market Hypothesis (EMH) in the context of the Algerian exchange rate market, focusing on the US Dollar, Euro, and British Pound exchange rate returns. Using tests for dependence, long memory, volatility clustering, and unit roots, the authors found that the ARMA–FIGARCH models best captured the behavior of exchange rate returns. When comparing these models to the Random Walk model for out-of-sample forecasting, the findings indicated predictability in exchange rate fluctuations, leading to a rejection of the EMH. Therefore, the results suggest that Algeria's exchange rate market is not fully efficient, allowing for potential forecasting and policy intervention opportunities.

Moreover, Godday et al (2022) investigated the persistence and long-run behavior of nominal exchange rates in Nigeria across three periods: Pre-GFC, Post-GFC, and the full sample. The study aimed to assess both the degree of persistence and the presence of fractional cointegration,

African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30, Juin 2025

applying the FCVAR model. The results showed that exchange rate persistence increased significantly in the Post-GFC period, underscoring the importance of enhanced coordination between fiscal and monetary authorities. Furthermore, while the CVAR model was more effective for the full sample, the FCVAR model proved superior in capturing the dynamics during both the Pre- and Post-GFC periods, indicating the presence of long-run relationships in Nigeria's nominal exchange rates.

Finally, El Mosallamy and Gamal (2024) investigated the impact of currency devaluation on the Egyptian stock market, focusing on the major devaluations of 2016 and 2022, and examined whether the market exhibited signs of efficiency. Using both parametric and non-parametric unit root tests, along with ARCH and EGARCH models applied to the EGX100 and EGX30 indices, the study tested the stationarity of returns. The results showed that returns remained stationary regardless of market events, suggesting a level of predictability in the market's behavior. These findings offer valuable insights for policymakers and investors seeking to evaluate and forecast market dynamics. The focus on Egypt was motivated by a lack of prior research and inconsistent conclusions in the existing literature on emerging markets, particularly Egypt.

Lastly, Jeffrey et al (2024) developed a new econometric framework to estimate and classify exchange rate regimes into four types: fixed, BBC (band, basket, and crawl), managed floating, and freely floating. The model captures both exchange rate dynamics and policy interventions under each regime. Special focus is given to the BBC regime, using a three-regime Threshold Auto Regressive (TAR) model to reflect its nonlinear behavior. To account for changes over time, the framework incorporates the Minimum Description Length (MDL) principle, addressing challenges of nonlinearity and structural breaks. Applied to 26 countries, the model effectively distinguishes regime behaviors and provides a robust tool for analyzing exchange rate systems.

Other studies have examined the Efficient Market Hypothesis using the multifractal approach. To begin with, Kyungsik and Seong-Min (2003) investigated the multifractal behavior of tick data in the Korean financial market, focusing on the won-dollar exchange rate and the KOSPI index. Using rescaled range (R/S) analysis, they confirmed the multifractal nature of returns and estimated both the standard Hurst exponent and the generalized q-th-order Hurst exponent within a universal multifractal framework. Their findings revealed that the market exhibits persistence and long-term memory effects. Additionally, crossovers in the Hurst exponents were observed at specific characteristic time scales. Unlike many financial series with fat-tailed

African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30, Juin 2025

distributions, the return distributions in their study were well described by a Lorentz distribution.

Similarly, Chiang et al (2010) investigated the multifractality and its sources in the returns of GBP/USD, EUR/USD, USD/JPY, and USD/CHF using Multifractal Detrended Fluctuation Analysis (MF-DFA). To identify the sources of multifractality, they applied shuffled and surrogate data using the Statically Transformed Autoregressive Process (STAP) method. The results showed that GBP/USD exhibited monofractal behavior, while EUR/USD, USD/JPY, and USD/CHF demonstrated multifractal characteristics. The fat tails in the return distributions explained multifractality in EUR/USD and USD/JPY, while both long memory and fat tails were responsible in USD/CHF. The study also found an unclear relationship between market liquidity and multifractality.

In addition, Stanislaw et al (2010) conducted a comprehensive analysis of high-frequency returns in the foreign exchange market using six currency pairs from two triangles: EUR–GBP–USD and GBP–CHF–JPY. They found that return fluctuations for all pairs were well modeled by q-Gaussian distributions under non-extensive statistical mechanics, with variations in the q-parameter reflecting the relative importance of each currency in global trading. Temporal correlations led to multifractal behavior across all exchange rates, with GBP/USD exhibiting the most symmetric singularity spectrum. They also analyzed triangular arbitrage residuals and discovered disproportionately heavy tails and anomalous multifractal properties, including negative singularity exponents and spectra, akin to those seen in turbulence and diffusion-limited aggregation. Cross-correlation analysis showed that inefficiencies at short time scales caused the Epps effect—delayed correlation buildup—over several hours, similar to stock markets, despite the higher liquidity and transaction frequency in forex markets. This suggests that the Epps effect arises from more than just transaction asynchronicity.

Moreover, Benbachir and El Alaoui (2011) performed Multifractal Detrended Fluctuation Analysis (MF-DFA) to investigate the multifractal properties of the Moroccan All Shared Index (MASI) and the Moroccan Most Active Shares Index (MADEX) from the Casablanca Stock Exchange (CSE). They calculated the generalized Hurst exponents, Rényi exponents, and singularity spectra for both indices. Using shuffling and phase randomization techniques, they identified two major sources of multifractality: long-range temporal correlations and fat-tail distributions. They found that long-range correlations primarily contributed to the multifractality of the MASI index, while both sources contributed almost equally for the MADEX index. Comparing the two, they observed that MASI exhibited richer multifractal

African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30, Juin 2025

behavior than MADEX, concluding that larger stock markets tend to have more complex dynamics, reflected in richer multifractal features. The study concluded that the Casablanca Stock Exchange is characterized by multifractal behavior.

Furthermore, Oh et al (2012) examined the multifractal properties of daily foreign exchange rates for Japan, Hong Kong, Korea, and Thailand relative to the U.S. dollar from 1991 to 2005. The analysis revealed that all four markets exhibited multifractal behavior. Comparing periods before and after the Asian currency crisis, the study found a notable increase in multifractality in the Korean and Thai markets, while Japan and Hong Kong showed more stable patterns. The increased multifractality was mainly attributed to extreme return values, highlighting the impact of financial shocks on market complexity.

Similarly, Harvey (2013) investigated the multifractality of the daily Philippine Peso–US Dollar exchange rate from January 2, 1998, to July 31, 2013, using multifractal detrended fluctuation analysis (MF-DFA). The analysis revealed clear multifractal characteristics in the exchange rate, with small fluctuations showing persistence. The study further determined that the primary source of this multifractality was the presence of broad fat-tail distributions in the data, rather than long-range correlations.

More recently, Chenyu et al (2019) examined the exchange rates of four major currencies—EUR, GBP, CAD, and JPY—against the USD from 2005 to 2019 using multifractal detrended fluctuation analysis (MF-DFA). They found that all four exchange rate series exhibited significant multifractal properties across the full time scale. Among them, the Japanese yen showed the lowest multifractality, indicating the highest market efficiency. The multifractal nature was attributed to long-range correlations and fat-tailed distributions. The authors also investigated cross-correlations between markets, revealing structural linkages. By dividing the sample into sub-periods based on the 2008 financial crisis and the 2014 U.S. quantitative easing withdrawal, they observed varying degrees of inefficiency in each period. Their findings challenged the Efficient Market Hypothesis (EMH) and provided both theoretical and practical insights into the dynamics, risks, and efficiency of foreign exchange markets.

Expanding the regional scope, Haouas (2021) conducted a multifractal analysis of exchange rates in the Middle East and North Africa (MENA) region using data from January 1999 to May 2017. The study aimed to assess the behavior of currency markets and test the efficiency hypothesis of the FOREX market in these countries. By estimating the scaling function and the Hölder exponent over time using the Generalized Quadratic Variation (GQV) method, the study found that all the examined exchange rate series exhibited multifractal characteristics.

African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30, Juin 2025

However, the degree of persistence varied across the different markets, suggesting differences in their efficiency levels.

Focusing on Poland, Katarzyna and Pietrych (2021) assessed the efficiency of the Polish zloty (PLN) exchange rate market using two methods: the Uncovered Interest Parity (UIP) framework and fractal analysis. Their findings showed that market efficiency holds only for the USD/PLN pair. For other currency pairs (EUR/PLN, JPY/PLN, CHF/PLN, MXN/PLN, and TRY/PLN), the UIP was rejected, indicating the presence of the forward premium anomaly and market inefficiency. Fractal analysis using the Hurst exponent further supported these results: most pairs exhibited long memory or anti-persistent (mean-reverting) behavior, while only USD/PLN had a Hurst exponent near 0.5, suggesting efficiency. Both methods consistently confirmed that USD/PLN is the only efficient market among the pairs studied.

Turning to Turkey, Ünal (2023) examined the multifractal behavior of hourly EUR/TRY and USD/TRY exchange rates from May 31, 2018, to March 21, 2022, using multifractal detrended fluctuation analysis (MF-DFA) and multifractal detrended cross-correlation analysis (MF-DCCA). The study identified multifractality in both individual exchange rates and their cross-correlations over the full period. A rolling window analysis revealed that the degree and sources of multifractality varied over time. These findings indicate that EUR/TRY and USD/TRY exchange rates exhibit complex, scale-dependent dynamics that traditional linear models cannot fully capture. The results underscore the need for more advanced modeling techniques for effective financial analysis, risk management, and trading strategies.

Moreover, Maruyama (2023) conducted a multifractal analysis using wavelet transform to examine the fractality and extreme value behavior of USD/JPY and EUR/JPY exchange rates. The study found that both exchange rates became multifractal following the 1997 Asian financial crisis. Over time, USD/JPY shifted to a monofractal and more stable pattern, correlating with yen depreciation, while EUR/JPY remained multifractal and unstable, also experiencing strong yen depreciation. After the 2007–2008 financial crisis, USD/JPY again exhibited monofractality and stability with yen appreciation, whereas EUR/JPY continued to show multifractality and instability, also with yen appreciation. Strong coherence between the two exchange rates was observed from 1995 to 2000. Using extreme value theory and Generalized Pareto (GP) models, the study predicted long-term return levels for both rates. Diagnostic tests validated the GP model fits, and the shape parameters suggested no finite upper limits for returns. For USD/JPY, the estimated 10-year and 100-year maximum return levels were 149.6 and 164.8, respectively, with corresponding 95% confidence intervals.

African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30, Juin 2025

Continuing with recent research, Datta (2024) analyzed daily exchange rates of the USD, GBP, EUR, and JPY against the Indian Rupee (INR) from 1999 to 2018 to investigate the presence and sources of multifractality. By examining the multifractal spectra of logarithmic returns and applying two transformations—random shuffling and phase randomization—the study aimed to identify whether multifractality stemmed from long-range correlations or fat tails. The findings revealed that all four currencies exhibited multifractal characteristics, but the sources varied: multifractality in USD returns was mainly due to fat tails; GBP and EUR were influenced by both fat tails and long-range correlations; and JPY was driven primarily by broad tails. These insights hold practical relevance for policymakers and investors in tailoring risk management and regulatory strategies.

2. Methodology

2.1. Data

This study employs daily closing exchange rate prices from Morocco, Algeria, and Tunisia, covering the period between January 1, 2007, and May 27, 2025, with a total of 4802 observations. The data were obtained from www.investing.com. The prices were subsequently transformed into logarithmic returns calculated as $r_t = ln\left(\frac{P_t}{P_{t-1}}\right) = ln(P_t) - ln(P_{t-1})$, where P_t represents the price and ln corresponds to the natural logarithm.

2.2. Method

In this section, we will present the MF-DFA method according to Kantelhardt et al. (2002). MF-DFA consists of five distinct steps.

Consider a time series $X = \left(X(k)\right)_{1\leq k\leq N}$, representing a financial series, where N is the length of the series. It is assumed that this series has a compact support, meaning that $X(k) = 0$ for only a negligible fraction of the values.

Step 1: We determine the profile $Y = \left(Y(i)\right)_{1\leq i\leq N}$ of the series X defined by:

$$Y(i) = \sum_{k=1}^{N} (X(k) - \bar{X}) \qquad (1)$$

where $\bar{X}$ is the mean of the series X.

Step 2: For a given time-scale s, we divide the profile Y into $N_S = Int(N/s)$ non-overlapping segments of the same length s, where $Int(.)$ represents the function that gives the integer part of a real number. Based on the recommendations of Peng et al. (1994), $5 \leq s \leq N/4$ is traditionally selected. Since N is generally not a multiple of s, a short part at the end of the

African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30, Juin 2025

profile may be neglected. To incorporate all the ignored parts of the series, the same procedure is repeated starting from the end of the profile. Thus, we obtain $2N_S$ segments and there are two types of segmentation: for $1 \leq i \leq s$

- $Y\big((v-1)s+\mathrm{i}\big)$ for $1 \leq v \leq N_S$.
- $Y\big((N-v-N_S)s+\mathrm{i}\big)$ for $N_S+1 \leq v \leq 2N_S$.

In each segment, we use the Ordinary Least Squares (OLS) method to properly fit the data with a local trend. In this study, we denote by $p_v^m(i)$ the fitting polynomial for the v-th segment: For $1 \leq v \leq 2N_S$

$$p_v^m(i) = \alpha_0^v + \alpha_1^v.i + \cdots + \alpha_m^v.i^m \tag{2}$$

In the empirical study, the order m can be quadratic, cubic, or even of a higher order. Choosing an appropriate value of m can avoid overfitting the series.

After determining $p_v^m(i)$, we calculate the variances $F^2(v,s)$ for all time scales s and for $1 \leq v \leq 2N_S$. The variance $F^2(v,s)$ is defined by:

$$F^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}\left[Y\big((v-1)s+i\big) - p_v^m(i)\right]^2 \tag{3}$$

for $1 \leq v \leq N_S$, and:

$$F^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}\left[Y\big((N-v-Ns)s+i\big) - p_v^m(i)\right]^2 \tag{4}$$

for $N_S+1 \leq v \leq 2N_S$.

Step 4: By averaging the variances over all segments, we obtain the fluctuation function $F_q(s)$ of order q defined by:

$$F_q(s) = \left[\frac{1}{2N_S}\sum_{i=1}^{2N_S}\big(F^2(v,s)\big)^{\frac{q}{2}}\right]^{\frac{1}{q}} \tag{5}$$

for $q \neq 0$, and:

$$F_0(s) = exp\left[\frac{1}{4N_S}\sum_{i=1}^{2N_S} ln\big(F^2(v,s)\big)\right] \tag{6}$$

for $q = 0$.

The purpose of the MF-DFA procedure is primarily to determine the behavior of the fluctuation functions $F_q(s)$ as a function of the time-scale s for various values of q. To this end, steps 2 through 4 must be repeated for different time scales s.

African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30, Juin 2025

Step 5: We analyze the multi-scale behavior of the fluctuation functions $F_q(s)$ by estimating the slope of the log-log plots of $F_q(s)$ versus s for different values of q. If the analyzed time series exhibits long-term correlation according to a power-law, such as fractal properties, the fluctuation function $F_q(s)$ will behave, for sufficiently large values of s, according to the following power-law scaling law:

$$F_q(s) \sim s^{H(q)} \qquad (7)$$

In general, the exponent $H(q)$ can depend on q. To estimate the values of $H(q)$ for different values of q, we perform a semi-logarithmic regression of the time series $H(q)$ on the time series $F_q(s)$.

If stationary time series are dealt with, only the exponen $H(2)$ is obtained, which is identically equal to the standard Hurst exponent H. Therefore, the exponent $H(q)$ generalizes the Hurst exponent H and is commonly referred to as the generalized Hurst exponent. To distinguish between monofractal and multifractal time series, we can say that if $H(q) = H$ for all values of q, then the studied time series is monofractal; otherwise, $H(q)$ is a monotonically decreasing function of q, and the corresponding time series is multifractal.

It is well known that the generalized Hurst exponent $H(q)$ is directly related to the multifractal scaling exponent $\tau(q)$, commonly known as the Rényi exponent:

$$\tau(q) = q.H(q) - 1 \qquad (8)$$

It is clear that monofractal time series are characterized by a linear form for the Rényi exponent:

$$\tau(q) = q.H - 1 \qquad (9)$$

where H is the Hurst exponent.

Another interesting way to characterize the multifractality of time series is to use the Hölder spectrum or the singularity spectrum $f(\alpha)$ of the Hölder exponent α. It is well known that the singularity spectrum $f(\alpha)$ is related to the Rényi exponent $\tau(q)$ through the Legendre transform:

$$\begin{cases} \alpha = \tau'(q) \\ f(\alpha) = q.\alpha - \tau(q) \end{cases} \qquad (10)$$

where $\tau'(q)$is the derivative of the function $\tau(q)$.

The richness of the multifractality can be determined by the width of the spectrum, $\Delta\alpha = \alpha_{max} - \alpha_{min}$. Thus, the wider the spectrum, the richer the multifractal behavior of the analyzed time series.

African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30, Juin 2025

We can easily deduce the relationship between the generalized Hurst exponent $H(q)$ and the singularity spectrum $f(\alpha)$:

$$\begin{cases} \alpha = h(q) + q.h'(q) \\ f(\alpha) = q.\big(\alpha - h(q)\big) + 1 \end{cases} \tag{11}$$

2.3. Sources of multifractality

Kantelhardt et al. (2002) identified two primary sources of multifractality in a time series, long-term temporal correlations and heavy-tailed distributions. To determine how each source contributes to the overall multifractality, we can use two transformations on the original return series, namely the shuffling (random permutation) and the surrogation (phase randomization). By shuffling the return series, the distribution of different moments is preserved, but long-term correlations are eliminated. After random permutation, the data have the same distribution but no temporal correlation or memory.

The surrogation helps isolate the contribution of long-term correlations to multifractality by randomly shifting the temporal phases of the data and disrupting these correlations while preserving their overall fluctuation behavior.

In the literature, there are various techniques for surrogation:

- Inverse Fast Fourier Transform (IFFT) (Proakis et al., 1996).
- Iterated Algorithm (iAAFT) (Schreiber and Schmitz, 1996).
- Statically Transformed Autoregressive Process (STAP) (Kugiumtzis, 2002)

In this study, we applied a shuffling technique called Randperm. For the surrogation we applied the Inverse Fast Fourier Transform (IFFT) method.

3. Results and discussion

This section starts by employing the MF-DFA method to investigate the multifractal characteristics of the return series for the Moroccan, Algerian, and Tunisian exchange rates. This is followed by an analysis aimed at pinpointing the underlying sources of the observed multifractality.

3.1. Multi-scale behavior of the fluctuation functions

Figure 1 shows the multi-scale behavior of the fluctuation functions $F_q(s)$ with respect to the time-scales s within the interval $S = [20:10:100{,}200:100:3000]$ for values of q in the interval $[-45:5:-5,-2.1:0.1:-0.1{,}0.1:0.1:2.1{,}5:5:45]$.

Figure N°1 : Fluctuation functions $Log\left(F_q^{XY}(s)\right)$ vs. $Log(s)$ for the three exchange rates

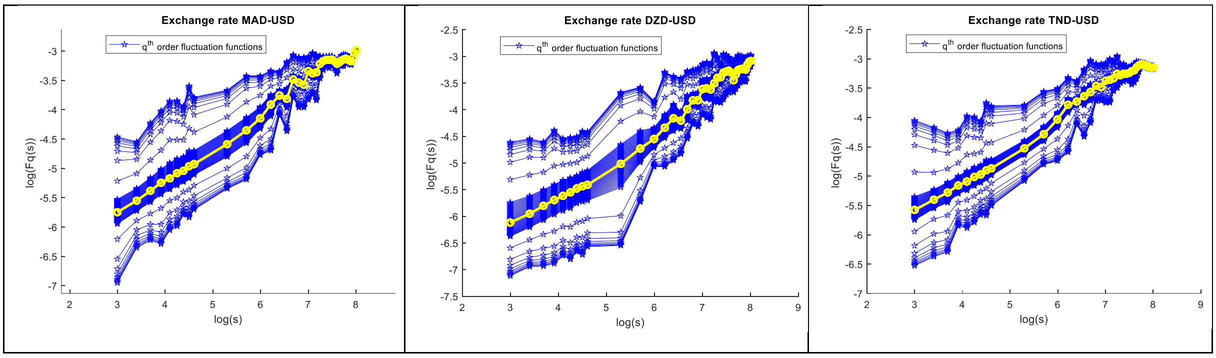


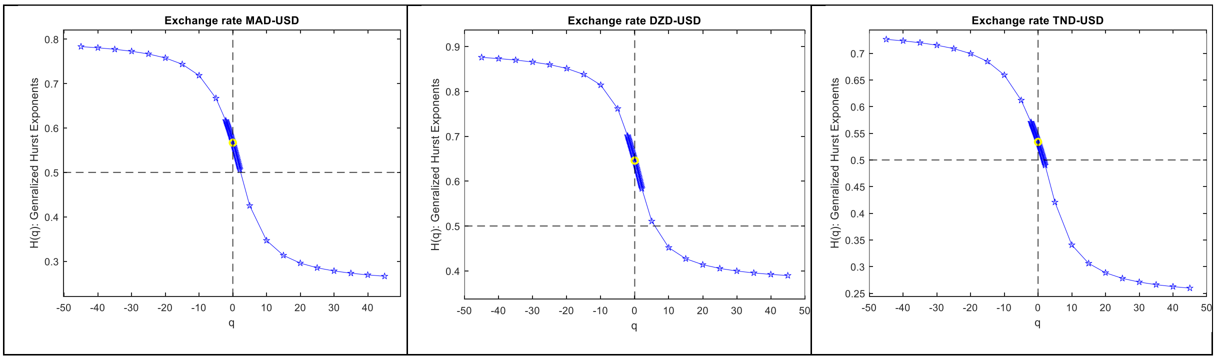
Figure 1 illustrates that for each exchange rate, the fluctuation functions follow distinct scaling behaviors across different time horizons and for various orders q, capturing both small (negative q) and large (positive q) fluctuations. The approximately linear trends in the double-logarithmic plots for each q suggest that the fluctuation functions exhibit power-law scaling with respect to time scale s, a hallmark of multifractal processes.

Furthermore, the divergence in slopes for different q values indicates the presence of multifractality in the exchange rate series. Specifically, the variation in scaling exponents across q highlights the heterogeneity of fluctuation intensities and the complex temporal structure embedded in the data. This supports the conclusion that the return series of the three exchange rates are governed by multifractal dynamics, influenced by both long-range correlations and extreme events.

3.2. Generalized Hurst Exponents $H(q)$

The generalized Hurst exponents $H(q)$ are given by the slopes of the lines obtained by least squares fitting $Log(s)$ on $Log\left(F_q(s)\right)$. Figure 2 shows the plots of the generalized Hurst exponents $H(q)$ as functions of $q \in [-45{:}5{:}-5, -2.1{:}0.1{:}-0.1, 0.1{:}0.1{:}2.1, 5{:}5{:}45]$.

Figure N°2 : Generalized Hurst exponents $H(q)$ for q in the interval $[-45{:}5{:}-5, -2.1{:}0.1{:}-0.1, 0.1{:}0.1{:}2.1, 5{:}5{:}45]$



African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30, Juin 2025

As shown in the previous figure, as q increases from -45 to 45, the generalized Hurst exponent $H(q)$ decreases non-linearly. This indicates that the three exchange rates exhibit multifractal nature, suggesting a weak form inefficiency of the three exchange rates markets.

The degree of multifractality in the returns series of the three exchange rates could be measured by the difference between the smallest and largest values of $H(q)$:

$$\Delta H = H_{max} - H_{min} = H(q_{min}) - H(q_{max}) \tag{13}$$

For a monofractal series, $\Delta H = 0$ is obtained. The larger ΔH is, the higher the degree of multifractality.

Table 1 presents the degree of multifractality for the three exchange rates, ordered in decreasing order based on the generalized Hurst exponent.

Table N°1 : Degrees of multifractality in decreasing order based on the generalized Hurst exponent

Rank	Exchange rate	ΔH
1	Morocco	0.499
2	Algeria	0.486
3	Tunisia	0.467

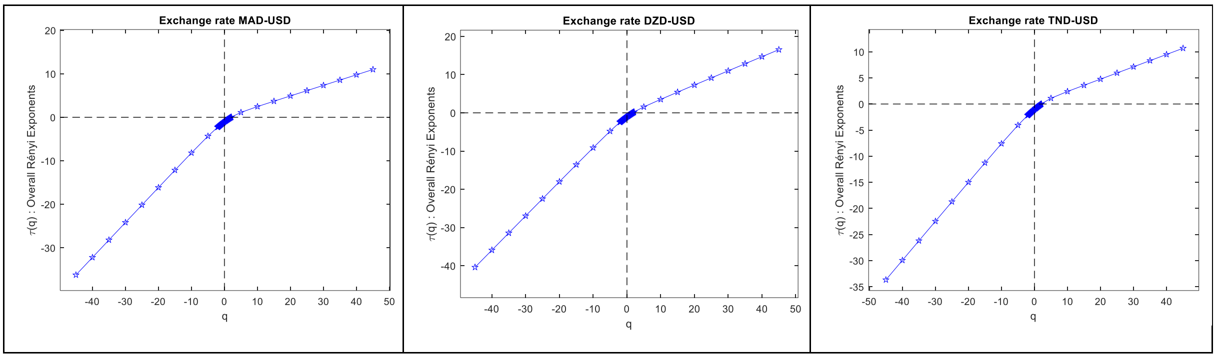
The ΔH value measures the extent of multifractality, with higher values indicating stronger multifractal behavior. According to table 1, the Moroccan exchange rate exhibits the highest degree of multifractality with a ΔH of 0.499, suggesting the most complex and heterogeneous scaling behavior among the three. The Algerian exchange rate follows closely with a ΔH of 0.486, indicating a slightly lower but still significant multifractal structure. The Tunisian exchange rate shows the lowest degree of multifractality at 0.467, implying relatively less complex dynamics compared to Morocco and Algeria.

Overall, this ranking suggests that the Moroccan exchange rate market demonstrates the greatest multifractal complexity, while Tunisia's market shows the least, with Algeria positioned in between.

3.3. Rényi Exponent $\tau(q)$

Figure 3 shows the plots of the Rényi exponents $\tau(q)$ as functions of the variable $q \in [-45{:}\,5{:}-5, -2.1{:}\,0.1{:}-0.1, 0.1{:}\,0.1{:}\,2.1, 5{:}\,5{:}\,45]$.

Figure N°3 : Rényi exponents $\tau(q)$ for
$q \in [-45:5:-5, -2.1:0.1:-0.1, 0.1:0.1:2.1, 5:5:45]$

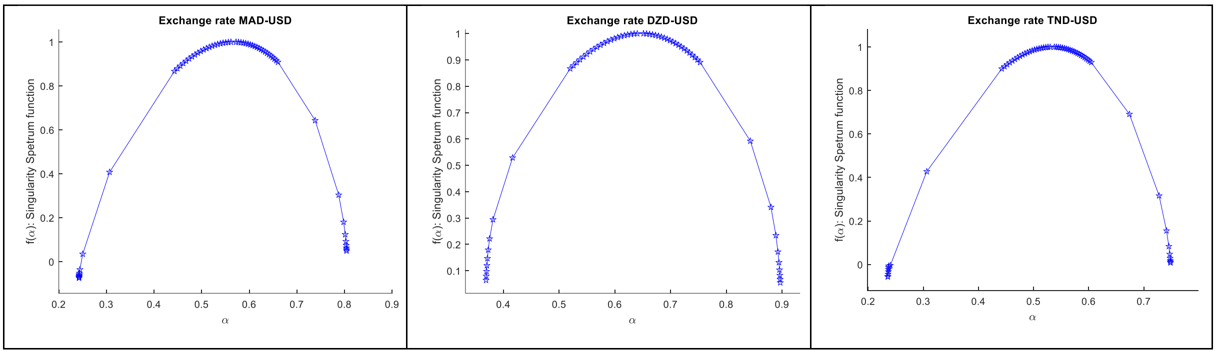


It can be observed in this figure that as q increases from -45 to 45, the Rényi exponent $\tau(q)$ increases non-linearly. This indicates that the three exchange rates exhibit multifractal nature, suggesting a weak form inefficiency in the three exchange rates markets.

3.4. Hölder singularity spectrum $f(\alpha)$

Another interesting way to characterize the multifractality of time series is to use the singularity spectrum $f(\alpha)$. Figure 4 shows the plots of the singularity spectrum functions $f(\alpha)$ for the six indices.

Figure N°4 : Singularity spectrum functions $f(\alpha)$ for the three exchange rates



We can observe in this figure that the curves of the singularity spectrum functions $f(\alpha)$ have an inverted parabolic shape. This indicates that the three exchange rates exhibit multifractal nature, suggesting a weak form inefficiency in the three exchange rates markets. Recall that for monofractal time series, the curve of the singularity spectrum theoretically reduces to a single point $\alpha = H$ with $f(\alpha) = 1$.

We can measure the degree of multifractality by calculating the width of the spectrum, given by:

$$\Delta\alpha = \alpha_{max} - \alpha_{min} \tag{14}$$

Table 2 presents the degree of multifractality for the three exchange rates in decreasing order.

Table N°2 : Degrees of multifractality in decreasing order based on the singularity spectrum

Rank	Exchange rate	$\Delta\alpha$
1	Morocco	0.562
2	Algeria	0.528
3	Tunisia	0.513

The singularity spectrum width $\Delta\alpha$ is a key measure of multifractality: larger values indicate a wider range of fractal exponents and therefore a stronger multifractal nature.

According to table 2, Morocco ranks first with the highest $\Delta\alpha$ of 0.562, indicating the most pronounced multifractal complexity among the three exchange rates. Algeria follows with a $\Delta\alpha$ of 0.528, showing a slightly less but still significant degree of multifractality. Tunisia has the lowest $\Delta\alpha$ at 0.513, suggesting it exhibits the least multifractal behavior in comparison to Morocco and Algeria.

3.5. Source of multifractality for the three exchange rates

As previously noted, there are two different sources of multifractality in a time series, long-term temporal correlations and the heavy tails distributions. To determine how each source contributes to the overall multifractality, we will use the shuffling and the surrogation transformations on the original geometric return series.

In this study, a shuffling technique is applied named “Randperm”. For the surrogation, the Inverse Fast Fourier Transform (IFFT) method is applied.

Figures 5 and 6 compare the curves of the generalized Hurst exponent $H(q)$ and the curves of the singularity spectrum $f(\alpha)$ for the three original series with those of the surrogate and the shuffled series.

Figure N°5 : Generalized Hurst exponent $H(q)$ vs. q for original, surrogate and shuffled

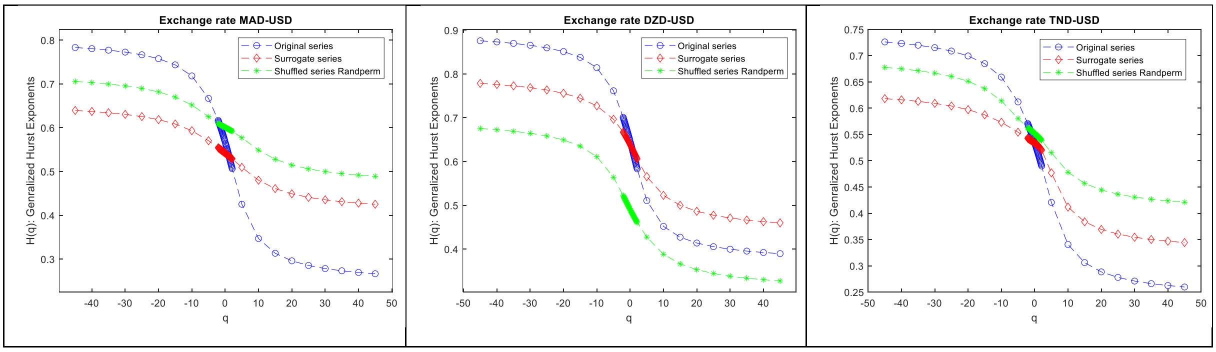
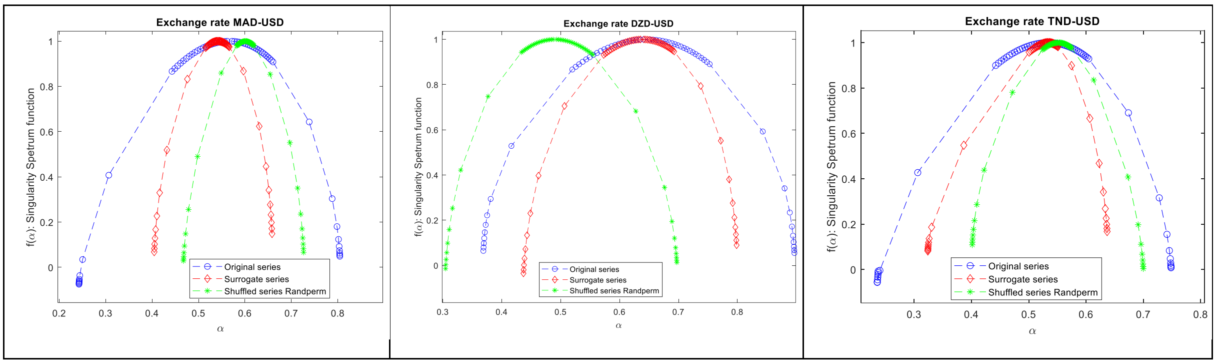


Figure N°6 : Singularity spectra $f(\alpha)$ vs. α for original, surrogate and shuffled



To precisely measure how much multifractality has been reduced, we calculate the values of $\Delta H = H_{max} - H_{min}$ and $\Delta\alpha = \alpha_{max} - \alpha_{min}$ for the three exchange.

The MF-DCCA program has been run 100 times for each index, and each time different results for the surrogate series and the shuffled series were obtained, the results for the original series remained consistent. This variability is due to the algorithms generating the surrogate and shuffled series using random permutations. However, in all 100 simulations, the ΔH and $\Delta\alpha$ of the original series are greater than the ΔH and $\Delta\alpha$ of the surrogate and the shuffled series. The results are presented in Table 3.

Table N°3 : Degrees of multifractality of original, surrogate and shuffled series based ΔH and $\Delta\alpha$

	Original		Surrogate		Shuffled-Randperm	
Exchange rate	ΔH	$\Delta\alpha$	ΔH	$\Delta\alpha$	ΔH	$\Delta\alpha$
Morocco	0.499	0.562	0.214	0.254	0.216	0.259
Algeria	0.486	0.528	0.319	0.362	0.348	0.393
Tunisia	0.467	0.513	0.274	0.313	0.257	0.299

The results show that for all three exchange rates, the values of ΔH and $\Delta\alpha$ in the original series are greater than those in both the surrogate and shuffled series (i.e., $\Delta H_{Originate} > \Delta H_{Surrogate}$, $\Delta\alpha_{orginale} > \Delta\alpha_{Surrogate}$, $\Delta H_{Originate} > \Delta H_{Shuffled}$ and $\Delta\alpha_{orginale} > \Delta H_{shuffled}$), as confirmed by the previous table. This demonstrates that multifractality is diminished by applying surrogation and shuffling procedures. Therefore, we conclude that the multifractal behavior observed in the three exchange rates arises from a combination of long-term correlations and heavy-tailed distributions.

The findings indicate that the multifractal behavior in the exchange rates of Morocco, Algeria, and Tunisia arises from both long-term correlations and heavy-tailed distributions. This has important practical implications: it suggests that traditional financial models assuming normality and independence may underestimate risks, highlighting the need for more sophisticated risk management tools that account for persistent patterns and extreme events. Traders can leverage these multifractal characteristics to improve forecasting and trading strategies, while investors should reconsider assumptions about market efficiency in these markets. Additionally, policymakers and regulators can use this understanding to better monitor and stabilize exchange rate markets, reducing systemic risks. Overall, these results emphasize the importance of adopting nonlinear and multifractal models for accurate analysis and decision-making in these foreign exchange markets.

3.6. Discussion

The empirical results of this study provide robust evidence of multifractal behavior in the exchange rate return series of Morocco, Algeria, and Tunisia, aligning with a growing body of literature that challenges the assumptions of the Efficient Market Hypothesis (EMH), particularly in emerging and developing economies.

Our findings, most notably the distinct scaling behavior of the fluctuation functions the nonlinear decline of the generalized Hurst exponent, and the inverted parabolic shapes of the singularity spectra, confirm that all three exchange rate series exhibit multifractal dynamics. These results are consistent with the studies by Chiang et al. (2010) and Chenyu et al. (2019), which identified strong multifractal properties in major currencies such as EUR/USD, GBP/USD, and USD/JPY. Similarly, Oh et al. (2012) found that the complexity of Asian currencies increased in the aftermath of financial crises, a dynamic which may also apply to North African currencies in light of regional economic instability.

The application of surrogate and shuffled data series confirms that the multifractality in all three markets is driven by a combination of long-range temporal correlations and heavy-tailed distributions. These findings are in line with the conclusions drawn by Kyungsik and Seong-Min (2003), Chiang et al. (2010), and Datta (2024), who similarly found that both temporal persistence and leptokurtic return distributions contribute to multifractal complexity. This dual-source explanation reinforces the notion that traditional linear models are inadequate for describing the dynamics of these exchange markets. Interestingly, the decomposition approach of Harvey (2013), who attributed Philippine Peso multifractality mainly to fat tails, contrasts with our results where both components significantly contribute. This distinction could be due to regional differences in market microstructure, regulatory environments, or levels of development.

African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30, Juin 2025

The presence of significant multifractal characteristics in all three exchange rates, along with evidence of long memory and extreme value effects, points to weak-form inefficiency. This directly contradicts earlier studies supporting EMH in various markets (e.g., Bogdan and Miloş, 2009; Amelot et al., 2017), but aligns with findings from Çıtak et al. (2015) in Turkey, Abounoori et al. (2012) in Iran, and Sashikanta and Pattanayak (2020) in India, where inefficiency was shown to vary over time and across regimes. The weak-form inefficiency observed in the Moroccan, Algerian, and Tunisian forex markets implies that historical price information retains some predictive power, and thus, these markets do not fully incorporate available information, a notion supported by the Adaptive Market Hypothesis (AMH) as discussed by Kumar (2018) and Njindan (2019).

African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30, Juin 2025

Conclusion

This study investigated the multifractal properties and efficiency of the Moroccan, Algerian, and Tunisian foreign exchange markets using the Multifractal Detrended Fluctuation Analysis (MF-DFA) method. The results provide strong evidence that all three exchange rate return series exhibit multifractal behavior, as demonstrated by the scale-dependent fluctuation functions, the non-linear behavior of the generalized Hurst exponents and the Rényi exponents, and the inverted parabolic shapes of the singularity spectra. These features reveal the presence of complex, heterogeneous temporal dynamics and indicate that none of the examined exchange markets conforms to the assumptions of the Efficient Market Hypothesis (EMH) in its weak form.

The degree of multifractality ranks Morocco as the most multifractal and thus the least efficient, followed by Algeria, and then Tunisia as the most efficient. These findings align with previous literature, including Benzai et al. (2022), which reported inefficiencies in the Algerian exchange market. However, our study reveals that Morocco demonstrates even greater inefficiency, likely due to more complex market structures or policy influences.

Furthermore, by comparing the original series with their surrogate and shuffled counterparts, we identified that the multifractal features arise from both long-range correlations and heavy-tailed distributions. This dual source of multifractality suggests that linear models assuming Gaussian behavior and independent returns are insufficient for modeling these markets accurately. Instead, nonlinear and multifractal frameworks are more appropriate.

Practically, the findings carry several implications. For traders and investors, the persistence and heterogeneity in returns imply the existence of patterns that can be exploited for strategy development, especially in less efficient markets like Morocco. Policymakers and regulators may also use these insights to improve transparency and market stability by addressing structural inefficiencies. Moreover, the variation in multifractality levels among the three markets emphasizes the need for tailored risk management and regulatory approaches.

A key limitation of this study is the inability to include exchange rate data from Egypt and Libya, both of which have faced significant political and economic challenges in recent years. These issues have hindered data availability, reliability, and consistency. As a result, the absence of data from these two countries restricts the scope of the analysis, preventing a broader comparison of multifractal behavior and market efficiency across the entire North African region. Including Egypt and Libya would have provided a more complete understanding of how exchange rate markets in these countries compare to those of Morocco, Algeria, and Tunisia.

African Scientific Journal
ISSN : 2658-9311
Vol : 03, Numéro 30, Juin 2025

A promising direction for future research is to explore cross-multifractal correlations between the foreign exchange markets of Morocco, Algeria, and Tunisia using Multifractal Detrended Cross-Correlation Analysis (MFDCCA). This method can capture the interdependencies and correlations between different markets, particularly in terms of their multifractal scaling behaviors. By applying MFDCCA, future studies could examine how fluctuations in one market affect the others, offering deeper insights into the complex dynamics and potential commonalities or differences in their behavior.

Additionally, future research could benefit from analyzing high-frequency data (such as minute-level or tick-by-tick data) to better capture short-term market dynamics. This approach would provide a more detailed understanding of the multifractal behavior and efficiency of the exchange rate markets at shorter time scales.

In conclusion, the multifractal behavior observed in the Moroccan, Algerian, and Tunisian exchange rates underscores the varying degrees of market inefficiency present in these markets, which are driven by both structural and statistical complexities. The presence of multifractality suggests that these markets do not follow the assumptions of the Efficient Market Hypothesis (EMH), where prices should reflect all available information and adjust instantaneously to new data. Instead, the markets display persistent, heterogeneous behaviors, indicating that inefficiencies persist over time, influenced by a range of factors.

At the structural level, inefficiencies in these markets may arise from factors such as market regulations, political instability, economic policies, and institutional frameworks that contribute to frictions, limit liquidity, or distort price discovery. For example, the exchange rate systems, governmental interventions, or central bank policies may lead to more complex dynamics than those predicted by traditional models, creating opportunities for deviations from efficiency.

On the statistical side, the nonlinear dynamics revealed by multifractal analysis suggest that the markets exhibit long-range correlations and heavy-tailed distributions that are inconsistent with the assumptions of random walks or Gaussian behavior typically used in traditional finance models. This indicates that the exchange rate returns in these countries do not behave in a simple or predictable manner, but instead display self-similar, scale-dependent patterns that require more sophisticated, nonlinear models to understand and forecast.

Thus, the multifractal behavior observed in the Moroccan, Algerian, and Tunisian markets provides important insights into their complexity and inefficiency, offering evidence that these markets may not be fully efficient in the weak form as outlined by EMH. This highlights the potential for investors, traders, and policymakers to better understand the underlying dynamics, develop tailored strategies, and implement reforms to improve market functioning, transparency, and stability.

REFERENCES

Amelot Marcelle, L., Ushad, S. and Lamport, M. (2017) Testing the Efficient Market Hypothesis in an Emerging Market: Evidence from Forex Market in Mauritius. *Theoretical Economics Letters*, 7, 2104-2122. doi: 10.4236/tel.2017.77143.

Abounoori, Esmaiel & Shahrazi, Mahdi & Rasekhi, Saeed (2012). "An investigation of Forex market efficiency based on detrended fluctuation analysis: A case study for Iran," Physica A: Statistical Mechanics and its Applications, Elsevier, vol. 391(11), pages 3170-3179.

Benbachir, Saâd & El Alaoui, Marwane, 2011. "A Multifractal Detrended Fluctuation Analysis of the Moroccan Stock Exchange International Research Journal of Finance and Economics ISSN 1450-2887 Issue 78 (2011) © EuroJournals Publishing, Inc. 2011. http://www.internationalresearchjournaloffinanceandeconomics.com

Benzai Yassine , Hadjar Soumia Aouad, Nassima Djellouli (2022). Long-Memory Models in Testing the Efficiency Market Hypothesis of the Algerian Exchange Market. December 2022. Management Dynamics in the Knowledge Economy 10(38):376-390. DOI:10.2478/mdke-2022-0024

Bogdan Dima and Laura Raisa Miloş (2009). TESTING THE EFFICIENCY MARKET HYPOTHESIS FOR THE ROMANIAN STOCK MARKET. Annales Universitatis Apulensis Series Oeconomica, 11(1),

Chenyu Han, Yiming Wang, Ye Ning (2019). Comparative analysis of the multifractality and efficiency of exchange markets: Evidence from exchange rates dynamics of major world currencies. Physica A: Statistical Mechanics and its Applications. Volume 535, 1. https://doi.org/10.1016/j.physa.2019.122365

Chiang, Shu-Mei & Lee, Yen-Hsien & Su, Hsin-Mei & Tzou, Yi-Pin, 2010. "Efficiency tests of foreign exchange markets for four Asian Countries," Research in International Business and Finance, Elsevier, vol. 24(3), pages 284-294, September.

Çıtak Levent, Veli Akel, and Murat Çetin (2015). Testing Random Walk Hypothesis in Turkish Foreign Exchange Market. Handbook of Research on Strategic Developments and Regulatory Practice in Global Finance. DOI: 10.4018/978-1-4666-7288-8.ch004

Datta Radhika Prosad (2024). Analysis of Indian foreign exchange markets : a multifractal detrended fluctuation analysis (mfdfa) approach. International journal of empirical economics.

- Singapore : World Scientific, ISSN 2810-9449, ZDB-ID 3142328-0. - Vol. 3.2024, 3, Art.-No. 2450006, p. 1-27

El Abed Riadh & Maktouf Samir, 2016. "Long Memory And Asymmetric Effect In East Asian Foreign Exchange Markets," Journal of Academic Research in Economics, Spiru Haret University, Faculty of Accounting and Financial Management Constanta, vol. 8(2 (July)), pages 294-306.

El Mosallamy Dalia Ahmed, Nesma Gamal (2024). Empirical Evidence on Testing the Efficient Market Hypothesis in Egypt: Case of Currency Devaluation. MSA-Management Sciences Journal. DOI:10.21608/msamsj.2024.257624.1049.

Fama, E.(1970). Efficient capital markets: A review of theory and empirical work. *The Journal of Finance, 25*(2), 383–417. https://doi.org/10.2307/2325486111.

Giannellis, Nikolaos & Papadopoulos, Athanasios P., 2009. "Testing for efficiency in selected developing foreign exchange markets: An equilibrium-based approach," Economic Modelling, Elsevier, vol. 26(1), pages 155-166, January.https://doi.org/10.1016/j.econmod.2008.06.013.

Godday U. Ebuh, Tersoo Shimonkabir Shitile and Nuruddeen Usman (2022). Contad. Adm vol.67 no.2 Ciudad de México abr./jun. 2022 Epub 10-Jul-2024. https://doi.org/10.22201/fca.24488410e.2022.3318

Haouas Nabiha, 2021. "Multifractal Analysis of the Foreign Exchange Markets Application to MENA Countries," Accounting and Finance Research, Sciedu Press, vol. 10(2), pages 1-17, May.

Harvey M. Niere (2013). Multifractality in the Philippine Foreign Exchange Market. Management Science and Engineering. Vol. 7, No. 3, 2013, pp. 67-70. DOI:10.3968/j.mse.1913035X20130703.2643

Jeffrey A. Frankel & Yao Hou & Danxia Xie, 2024. "Estimation of Nonlinear Exchange Rate Dynamics in Evolving Regimes," NBER Working Papers 32644, National Bureau of Economic Research, Inc.

Kallianiotis Ioannis N., 2017. "Tests of Efficiency in the Foreign Exchange Market," International Journal of Economics and Financial Research, Academic Research Publishing Group, vol. 3(10), pages 218-239, 10-2017.

Kantelhardt, J.W., Stephan, A.Z., Koscielny-Bunde, E., Shlomo, H., Bunde, A., Stanley, H.E. (2002). Multifractal detrended fluctuation analysis of nonstationary time series. Physica A Statistical Mechanics and its Applications, 316 (1–4), 15 87-114. DOI:10.1016/S0378-4371(02)01383-3.

Katarzyna Czech and Łukasz Pietrych (2021). The Efficiency of the Polish Zloty Exchange Rate Market: The Uncovered Interest Parity and Fractal Analysis Approaches. *Risks* 2021, *9*(8), 142; https://doi.org/10.3390/risks9080142.

Kugiumtzis, D. (2002). Statically transformed autoregressive process and surrogate data test for nonlinearity. *Physical Review E*.

Kumar, D. (2018) Market Efficiency in Indian Exchange Rates: Adaptive Market Hypothesis. *Theoretical Economics Letters*, 8, 1582-1598. doi: 10.4236/tel.2018.89101.

Kyungsik Kim & Seong-Min Yoon, 2003. "Multifractal Features in the Foreign Exchange and Stock Markets," Papers cond-mat/0305270, arXiv.org.

Maruyama Fumio (2023). Analysis of the USD/JPY and EUR/JPY Exchange Rates Using Multifractal Analysis and Extreme Value Theory. Journal of Applied Mathematics and Physics > Vol.11 No.10, October 2023. DOI: 10.4236/jamp.2023.1110184

Namhoon Lee, Wonseok Choi, Yuntaek Pae (2021). Market efficiency in foreign exchange market. Economics Letters. Volume 205, 109931. https://doi.org/10.1016/j.econlet.2021.109931

Njindan Bernard Iyke, 2019. "A Test Of The Efficiency Of The Foreign Exchange Market In Indonesia," Bulletin of Monetary Economics and Banking, Bank Indonesia, vol. 21(12th BMEB), pages 439-464, January.

Oh G. , C. Eom, S. Havlin, W. -S. Jung, F. Wang, H. E. Stanley & S. Kim (2012). A multifractal analysis of Asian foreign exchange markets. The European Physical Journal B. Volume 85, article number 214. DOI:10.1140/epjb/e2012-20570-0.

Peng, C.K., Buldyrev, S.V., Havlin, S., Simons, M., Stanley, H.E., Goldberger, A.L. (1994). Mosaic organization of DNA nucleotides. Physical Review E, 49, 1685. DOI:10.1103/PhysRevE.49.1685.

Proakis, J.G., & Dimitris, G.M. (1996). Digital Signal Processing: Principles, Algorithms, and Applications. *Publisher: Prentice Hall.*

Sashikanta Khuntia & J. K. Pattanayak, 2020. Evolving Efficiency of Exchange Rate Movement: An Evidence from Indian Foreign Exchange Market, Global Business Review, International Management Institute, vol. 21(4), pages 956-969.

Schreiber, T., Schmitz, A. (1996). Improved Surrogate Data for Nonlinearity Tests. *Physical Review Letters*, Volume 77, Number 4.

Stanislaw Drozdz & Jaroslaw Kwapien & Pawel Oswiecimka & Rafal Rak, 2010. "The foreign exchange market: return distributions, multifractality, anomalous multifractality and Epps effect," New Journal of Physics 12 (2010) 105003 (23pp).

Ünal, B. (2023). Time-Varying Fractal Analysis of Exchange Rates. Chaos Theory and Applications, 5(3), 242-255. https://doi.org/10.51537/chaos.1305009. https://doi.org/10.51537/chaos.1305009.